Newton's method.

1. 原始牛顿法：求方程的根：如：$x^3-2x-5=0$，$e^x+x=0$，$\cos x = x$ $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

2. 最优化中的牛顿法：求解无约束优化问题. $x_{n+1} = x_n - [\nabla^2 f(x_n)]^{-1} \nabla f(x_n)$

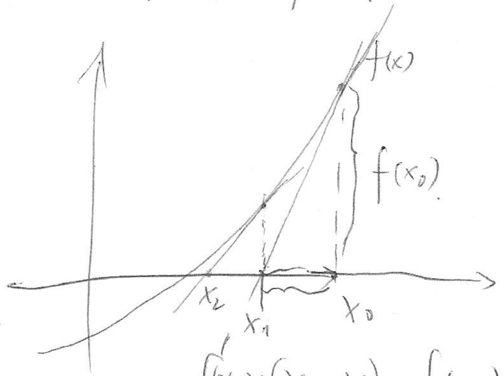
例子：$f(x) = x^2 - 2 = 0$

$$x_{n+1} = x_n - \frac{x_n^2-2}{2x_n} = \frac{x_n + \frac{2}{x_n}}{2}$$

$x_0 = 1$

$x_1 = 1.5$

$x_2 = 1.41667 \ldots$



$$f'(x_0)\cdot(x_0 - x_1) = f(x_0)$$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

The goal of Newton's method for estimating a solution of an equation $f(x)=0$ is to produce a sequence of approximations that approach a solution.